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Macroeconomic Theory
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A New-Keynesian Model

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You were introduced to a monopolistic competition model with price rigidities in class where only a constant fraction of firms, the so-called flexible-price firms, could change their prices after the realization of a shock; the rest had to set their prices at the beginning of each period and keep their promise of delivering the demand after the realization of exogenous shocks.

Today, we are going to present a similar basic New-Keynesian model in which monopolistic firms can adjust their prices in each period with a constant probability of $1-\theta$. This is based on the staggered price-setting model of Calvo (1983). Thus, after the price is set, the expected duration of this price being effective is $1/(1-\theta)$. This creates a *so-called* (‼) micro-founded rigidity in aggregate price level.

To make this interesting, imagine there is a fairy in this economy, which we refer to as *Calvo fairy!* This fictitious being appears at the beginning of each period t, chooses fraction $(1-\theta)$ of firms randomly and taps them on the shoulder, giving them the permission to change their prices. This story occurs in every period.

A more *realistic* story is that there are small costs of changing prices in this economy, referred to as *menu costs*. So that, in each period, only a fraction θ of firms find it too costly to adjust their prices to unanticipated shocks in economy. We will invite you to think about the difference of these two settings, in terms of persistence of unanticipated nominal shocks in the economy!

For a firm that sets its price in period t, there is a $(1-\theta)$ chance of resetting price next period. This means there is a $(1-\theta)$ probability of the price in t being in effect only in period t, without experiencing any rigidity in price. With probability θ, the firm has to keep its price fixed in period $t+1$. So, the chance of a one-period price rigidity is $\theta(1-\theta)$. Therefore, the expected length of price rigidity is

$$\sum_{k=0}^{\infty} \theta^k (1-\theta) k,$$

which is a geometric power series converging to $(1-\theta)^{-1}$.

We are going to study the implications of such assumptions for the neutrality of monetary policy and nominal shocks in such an economy.

Households

The economy is inhabited by an infinitely-lived representative household whose preferences over streams of consumption and labor are represented by the following utility function with expected form:

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right),$$

where C_t is a consumption index, given by

$$C_t \coloneqq \left[\int_0^1 C_t\left(i\right)^{\frac{\epsilon-1}{\epsilon}}\, di\right]^{\frac{\epsilon}{\epsilon-1}}.$$

Here, $C_t\left(i\right)$ is the consumption of a differentiated good indexed by i. We will assume there is an exogenous continuum of such goods in the economy, of unit mass.

The budget constraint of the household is given by

$$\int_0^1 P_t\left(i\right) C_t\left(i\right) di + Q_t B_t \leq B_{t-1} + W_t N_t + T_t,$$

where $p_t\left(i\right)$ is the price of variety i, B_t is a one-period risk free bond with face value price Q_t, W_t is the wage rate, and T_t is the lump-sum transfers to the households (possibly in terms of profits, etc.).

Thus, we may write the problem of the household as:

$$\begin{aligned}
\max \quad & E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right) \\
s.t. \quad & C_t = \left[\int_0^1 C_t\left(i\right)^{\frac{\epsilon-1}{\epsilon}}\, di\right]^{\frac{\epsilon}{\epsilon-1}} \\
& \int_0^1 P_t\left(i\right) C_t\left(i\right) di + Q_t B_t \leq B_{t-1} + W_t N_t + T_t \\
& \lim_{T\to\infty} E_t\left(B_t\right) \geq 0,
\end{aligned}$$

where the last condition rules out the possibility of Ponzi schemes in the economy.

Exercise 1 *What is the difference between condition* $\lim_{T\to\infty} E_t(B_t) \geq 0$*, and the no-Ponzi scheme condition you have seen previously (e.g. in* Tim*'s course)? Which one is more strict?*

We can think of a household's problem in period t as consisting of two parts: deciding about the fraction of wealth household wants to allocate to consumption, and dividing this fraction between different varieties of consumption goods. We begin with the latter; i.e. given amount $\bar{W}$ of wealth dedicated to consumption and price of each differentiated good, how would household divide $\bar{W}$ between varieties.

Given a constant amount of wealth, $\bar{W}$, spent on consumption in period t,

$$\int_0^1 P_t(i)\, C_t(i)\, di = \bar{W},$$

we can write the problem of the household as choosing between the different varieties as:

$$\begin{aligned} \max_{C(i)} \quad & U(C, N) \\ s.t. \quad & C = \left[\int_0^1 C(i)^{\frac{\epsilon-1}{\epsilon}}\, di\right]^{\frac{\epsilon}{\epsilon-1}} \\ & \int_0^1 P(i)\, C(i)\, di = \bar{W}. \end{aligned}$$

The first order condition for the optimal consumption of variety i is:

$$U_C(C, N)\, C(i)^{\frac{\epsilon-1-\epsilon}{\epsilon}} \left[\int_0^1 C(k)^{\frac{\epsilon-1}{\epsilon}}\, dk\right]^{\frac{\epsilon-\epsilon+1}{\epsilon-1}} = \lambda P(i), \quad \forall i \in [0, 1],$$

where λ is the Lagrange multiplier on the constraint of the problem. By writing the same condition for variety j, and dividing the two equations, we have:

$$\frac{C(i)^{-1/\epsilon}}{C(j)^{-1/\epsilon}} = \frac{P(i)}{P(j)},$$

$$\therefore C(i) = \left[\frac{P(j)}{P(i)}\right]^{\epsilon} C(j).$$

If we substitute this result into the definition of consumption index, we get:

$$C = \left[\int_0^1 \left[\left[\frac{P(j)}{P(i)}\right]^{\epsilon} C(j)\right]^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}},$$
$$\therefore C = \left[\int_0^1 \left[\frac{P(j)}{P(i)}\right]^{\epsilon-1} C(j)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}},$$

and, hence:

$$C(j) = \frac{P(j)^{\epsilon}}{\left[\int_0^1 P(i)^{1-\epsilon} di\right]^{\frac{\epsilon}{\epsilon-1}}} C,$$
$$\therefore C(j) = \left[\frac{P(j)}{P}\right]^{-\epsilon} C, \qquad (1)$$

where:

$$P \coloneqq \left[\int_0^1 P(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}. \qquad (2)$$

We will refer to Equation (1) as the demand function for variety j, as a function of its price $P(j)$, aggregate consumption index C, and aggregate price index P.

To see the intuition behind the aggregate price index, let us substitute this result back into the constraint of the optimization problem above, to get:

$$\begin{aligned}
\bar{W} &= \int_0^1 P(i)\, C(i)\, di \\
&= \int_0^1 P(i) \left[\frac{P(i)}{P}\right]^{-\epsilon} C di \\
&= \frac{\int_0^1 P(i)^{1-\epsilon} di}{\left[\int_0^1 P(i)^{1-\epsilon} di\right]^{\frac{-\epsilon}{1-\epsilon}}} C \\
&= \left[\int_0^1 P(i)^{1-\epsilon} di\right]^{1+\frac{\epsilon}{1-\epsilon}} C \\
&= P.C.
\end{aligned}$$

Therefore, we may write the first part of a household's problem simply as:

$$\begin{aligned} \max \quad & E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \\ s.t. \quad & P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + T_t \\ & \lim_{T \to \infty} E_t(B_t) \geq 0, \end{aligned}$$

where P_t is defined in (2). The first order conditions for this problem are:

1. $\beta^t U_C(C_t, N_t) = P_t \lambda_t$,
2. $\beta^t U_N(C_t, N_t) = -W_t \lambda_t$,
3. and $Q_t \lambda_t = \sum_{s_{t+1}} \lambda_{t+1}(s_{t+1})$ (where s_{t+1} is an index for next period's state).

If we combine conditions 1 and 2, we get the intratemporal optimality condition for labor supply:

$$-\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} = \frac{W_t}{P_t}. \tag{3}$$

Moreover, we can derive the Euler equation by combining 2 and 3:

$$Q_t = \beta E_t \left(\frac{U_C(C_{t+1}, N_{t+1})}{U_C(C_t, N_t)} \frac{P_t}{P_{t+1}} \right). \tag{4}$$

Remark *To see how we can derive this, assume for the moment that next period's state is given by s_t. Then:*

$$\begin{aligned} Q_t \frac{\beta^t U_C(C_t, N_t)}{P_t} &= \sum_{s_{t+1}} \frac{\beta^{t+1} U_C(C_{t+1}(s_{t+1}), N_{t+1}(s_{t+1}))}{P_{t+1}(s_{t+1})} \\ &= \beta^{t+1} E_t \frac{U_C(C_{t+1}, N_{t+1})}{P_{t+1}}. \end{aligned}$$

(4) *can be derived using this equality.*

Now, if we assume that the period utility takes the following separable form,

$$U(C, N) = \frac{C^{1-\sigma}}{(1-\sigma)} - \frac{N^{1+\phi}}{(1+\phi)},$$

then, we can write (4) as:

$$1 = E_t \left(\beta \frac{1}{Q_t} \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} \right),$$

which can be written in natural log terms as:

$$1 = E_t \left(\exp \left(\log \beta + i_t - \sigma \Delta c_{t+1} - \pi_{t+1} \right) \right). \tag{5}$$

The lower case letters indicate the logarithm of the variables. i_t is defined as $\log(1/Q_t)$.

Remark i_t *is the* net nominal rate of return. *To see why, note that the gross nominal rate of return is* $1/Q_t$*. If we denote this by* $1 + i_t$*, then:*

$$\begin{aligned} \log(1 + i_t) =& \log\left(\frac{1}{Q_t}\right) \\ \simeq& i_t. \end{aligned}$$

On a balanced growth path, where consumption grows at a constant rate γ, we have:

$$i = -\log \beta + \sigma\gamma + \pi.$$

A first order Taylor expansion of (5) around the balanced growth path yields:

$$\exp \left(\log \beta + i_t - \sigma \Delta c_{t+1} - \pi_{t+1} \right) \simeq 1 + \log \beta + i_t - \sigma \Delta c_{t+1} - \pi_{t+1}.$$

If we substitute this result into Equation (5), we get a first order log-linearization of this condition:

$$\begin{aligned} \log \beta + i_t - \sigma E_t \left(c_{t+1} - c_t \right) - E_t \left(\pi_{t+1} \right) = 0, \\ \therefore c_t = E_t \left(c_{t+1} \right) - \frac{1}{\sigma} \left[\log \beta + i_t - E_t \left(\pi_{t+1} \right) \right]. \end{aligned} \tag{6}$$

On the other hand, under the assumption of separable utility form, we may write Equation (3) as:

$$\frac{N_t^{\phi}}{C_t^{-\sigma}} = \frac{W_t}{P_t}.$$

In logarithmic terms, we can rewrite this as:

$$w_t - p_t = \sigma c_t + \phi n_t. \tag{7}$$

To introduce money explicitly in the model, we add an ad-hoc log-linear money demand equation to the household side of the economy as well:

$$m_t - p_t = y_t - \eta i_t.$$

This equation, determines the path of nominal interest rate i_t as a function of exogenous money supply M_t in the equilibrium.

To see the intuition behind this money demand function, consider the following problem faced by a household that values money holdings (we have seen how such a utility form may emerge from a monetary economy):

$$\begin{aligned} \max \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{(1-\sigma)} + \frac{(M_t/P_t)^{1-\nu}}{(1-\nu)} - \frac{N_t^{1+\phi}}{(1+\phi)} \right] \\ s.t. \quad & P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t + T_t \\ & \lim_{T \to \infty} E_t (B_t) \geq 0. \end{aligned}$$

The first order conditions for the household are:

1. $\beta^t C_t^{-\sigma} = P_t \lambda_t$,
2. $\beta^t \frac{1}{P_t} \left(\frac{M_t}{P_t} \right)^{-\nu} = \lambda_t - \lambda_{t+1}$,
3. and $Q_t \lambda_t = \lambda_{t+1}$.

If we substitute from 3 and 1 into 2, we get:

$$\begin{aligned} \beta^t \frac{1}{P_t} \left(\frac{M_t}{P_t} \right)^{-\nu} &= (1 - Q_t) \beta^t \frac{1}{P_t} C_t^{-\sigma}, \\ \therefore \left(\frac{M_t}{P_t} \right)^{-\nu} &= (1 - Q_t) C_t^{-\sigma}. \end{aligned}$$

Using our definition of nominal rate of return, we may write this condition as:

$$\frac{(M_t/P_t)^{-\nu}}{C_t^{-\sigma}} = 1 - e^{-i_t} \simeq i_t.$$

Now, if we take the natural logarithm, we may write this equation as follows:

$$\begin{aligned} m_t - p_t =& \frac{\sigma}{\nu} c_t - \frac{1}{\nu} \log\left(1 - e^{-i_t}\right) \\ \simeq& \frac{\sigma}{\nu} c_t - \frac{1}{\nu\left(e^{i_t} - 1\right)} i_t. \end{aligned}$$

Assuming unit elasticity of (real) money demand with respect to consumption (i.e. $\sigma/\nu = 1$) and incorporating market clearing condition, then, imply:

$$m_t - p_t = y_t - \eta i_t.$$

Firms

There is a continuum of monopolistically competitive firms in the economy that produce a differentiated good according to the same technology, given by:

$$Y_t(i) = A_t N_t(i).$$

Firm i faces a demand function of the form in (1), and takes the aggregate price index P_t and aggregate consumption index C_t as given.

In each period, fraction $1-\theta$ of the firms have the chance to adjust their prices (while fraction θ has to keep their price fixed and meet the demand in the market). Therefore, the average interval between price changes for a firm is $1/(1-\theta)$. In this sense, θ is an index of price stickiness in this economy.

The problem faced by a firm that is chosen in period t to adjust its price can be written as:

$$\begin{aligned} \max_{p_t^*} \quad & \sum_{k=0}^{\infty} \theta^k E_t \left(Q_{t,t+k} \left(P_t^* Y_{t+k|t} - \Psi_{t+k} \left(Y_{t+k|t} \right) \right) \right) \\ s.t. \quad & Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k}, \end{aligned}$$

where Ψ is the cost function. $Q_{t,t+k}$ is the stochastic discount factor; it determines how future nominal profits are discounted in date t's utility terms, and can be evaluated as:

$$\begin{aligned} Q_{t,t+k} =& E_t \left(\prod_{s=0}^{k-1} Q_{t+s} \right) \\ =& E_t \left(\prod_{s=0}^{k-1} \beta E_{t+s} \left(\frac{U_C \left(C_{t+s+1}, N_{t+s+1} \right)}{U_C \left(C_{t+s}, N_{t+s} \right)} \frac{P_{t+s}}{P_{t+s+1}} \right) \right) \\ =& \beta^k E_t \left(\frac{U_C \left(C_{t+k}, N_{t+k} \right)}{U_C \left(C_t, N_t \right)} \frac{P_t}{P_{t+k}} \right), \end{aligned}$$

where the second equality follows from the *Law of Iterated Expectations.*

It is clear that any firm tapped by the Calvo fairy to adjust its price would face the same problem in period t. Therefore, all the firms adjusting prices in period t would choose

If we substitute for $Y_{t+k|t}$ into the objective function, we can rewrite the problem of the monopolist as:

$$\max_{P_t^*} \quad \sum_{k=0}^{\infty} \theta^k E_t \left(Q_{t,t+k} \left(\left(P_t^* \right)^{1-\epsilon} P_{t+k}^{\epsilon} C_{t+k} - \Psi_{t+k} \left(\left(P_t^* \right)^{-\epsilon} P_{t+k}^{\epsilon} C_{t+k} \right) \right) \right)$$

The first order condition for this problem, with respect to the control P_t^* yields::

$$\sum_{k=0}^{\infty} \theta^k E_t \left(Q_{t,t+k} \left(\left(1 - \epsilon \right) \left(P_t^* \right)^{-\epsilon} P_{t+k}^{\epsilon} C_{t+k} + \epsilon \left(P_t^* \right)^{-\epsilon-1} P_{t+k}^{\epsilon} C_{t+k} \frac{\partial \Psi_{t+k} \left(Y_{t+k|t} \right)}{\partial Y_{t+k|t}} \right) \right) = 0.$$

If we substitute back $Y_{t+k|t}$, we get:

$$\sum_{k=0}^{\infty} \theta^k E_t \left(Q_{t,t+k} \left(\left(1 - \epsilon \right) Y_{t+k|t} + \epsilon \left(P_t^* \right)^{-1} Y_{t+k|t} \frac{\partial \Psi_{t+k} \left(Y_{t+k|t} \right)}{\partial Y_{t+k|t}} \right) \right) = 0.$$

If we multiply both sides of the equality by $P_t^* / (1-\epsilon)$, we have:

$$\sum_{k=0}^{\infty} \theta^k E_t \left(Q_{t,t+k} Y_{t+k|t} \left(P_t^* - \frac{\epsilon}{(\epsilon-1)} \psi_{t+k|t} \right) \right) = 0,$$

where $\psi_{t+k|t} \coloneqq \partial \Psi_{t+k} \left(Y_{t+k|t} \right) / \partial Y_{t+k|t}$ is the nominal marginal cost of the firm in period $t+k$.

Note that, when $\theta = 0$, this condition gives the familiar frictionless optimal pricing condition of

$$P_t^* = \frac{\epsilon}{(\epsilon-1)} \frac{\partial \Psi_{t+k} \left(Y_{t+k|t} \right)}{\partial Y_{t+k|t}};$$

i.e. price of the monopolistic firm is marked up above the marginal cost, by a constant that depends on the elasticity of demand. We denote this *friction-less mark-up* by $\mathscr{M}$.

If we divide this equation be P_{t-1}, we get:

$$\sum_{k=0}^{\infty} \theta^k E_t \left(Q_{t,t+k} Y_{t+k|t} \left(\frac{P_t^*}{P_{t-1}} - \mathscr{M} \frac{\psi_{t+k|t}}{P_{t+k}} \frac{P_{t+k}}{P_{t-1}} \right) \right) = 0.$$

By letting $P_{t+k}/P_{t-1} \coloneqq \Pi_{t-1,t+k}$ be the inflation between periods $t-1$ and $t+k$, we can rewrite this condition as:

$$\sum_{k=0}^{\infty} \theta^k E_t \left(Q_{t,t+k} Y_{t+k|t} \left(\frac{P_t^*}{P_{t-1}} - \mathscr{M} MC_{t+k|t} \Pi_{t-1,t+k} \right) \right) = 0, \tag{8}$$

where $MC_{t+k|t}$ is the *real marginal cost* in period $t+k$ of a firm that sets its price at t.

Consider a steady-state of this economy in which the inflation rate is zero, $\Pi_{t-1,t+k} = 1$; the price level remains constant, and, hence, $P_t^* = P_t = P_{t-1}$. Moreover, $C_t = Y_t = Y_{t+k|t}$ remains a constant, and $Q_{t,t+k} = \beta^k$. In addition,

$$\frac{P_t^*}{P_{t-1}} - \mathscr{M} MC_{t+k|t} \Pi_{t-1,t+k} = 0,$$

from firm's first order condition. Therefore:

$$MC_{t+k|t} = \frac{1}{\mathscr{M}},$$

so that, in the zero-inflation steady-state, the real marginal costs of all firms are equal and constant.

We can log-linearize (8) around this *zero-inflation steady-state* as:

$$p_t^* - p_{t-1} = (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left(\widehat{mc}_{t+k|t} + p_{t+k} - p_{t-1}\right), \tag{9}$$

where

$$\begin{aligned} \widehat{mc}_{t+k|t} &:= mc_{t+k|t} - mc \\ &= mc_{t+k|t} + \log \mathscr{M}, \end{aligned}$$

is the log deviation of marginal cost in $t+k$ from its steady-state value, mc.

Remark *To log-linearize an equation of the form $f(X_t, Y_t) = g(Z_t)$ around the steady-state, use the following formula:*

$$[f_1(X,Y) X x_t + f_2(X,Y) Y y_t] \simeq g'(Z) Z z_t,$$

where X,Y, and Z are the steady-state values of X_t, Y_t, and Z_t, and small letters denote the natural logarithms.

Aggregate Price Level Dynamics

Let $S_t \subset [0,1]$ be the firms that are not adjusting their prices in period t. The aggregate price index in period t, as we defined it before, is then given by:

$$\begin{aligned} P_t^{1-\epsilon} &= \int_0^1 P_t(i)^{1-\epsilon} \, di \\ &= \int_{S_t} P_{t-1}(i)^{1-\epsilon} \, di + \int_{S_t^c} P_t(i)^{1-\epsilon} \, di. \end{aligned}$$

Noting that each firm that is allowed to adjust its price would choose P_t^* in t, and the measure of firms that can adjust is $1-\theta$, we can write:

$$\begin{aligned}\int_{S_t^c} P_t(i)^{1-\epsilon}\, di &= (P_t^*)^{1-\epsilon}\int_{S_t^c} di \\ &= (1-\theta)(P_t^*)^{1-\epsilon}.\end{aligned}$$

On the other hand:

$$\begin{aligned}\int_{S_t} P_{t-1}(i)^{1-\epsilon}\, di =& \theta\int_0^1 P_{t-1}(i)^{1-\epsilon}\, di \\ =&\theta P_{t-1}^{1-\epsilon}.\end{aligned}$$

To see why this is the case, consider the firms all with their prices equal to $\bar{P}_{t-1}$; suppose these firms make a set od measure di. Of this set, exactly fraction θ of firms cannot adjust their price in t. Therefore, a set of measure θdi of firms would have prices equal to $\bar{P}_{t-1}$ in t. These firms constitute fraction $\theta di/\mu(S_t)$ of the set S_t (where μ is the measure of set S_t). If we repeat this argument for all the prices that prevail in $t-1$, we end up covering the entire set S_t. Thus:

$$\int_{S_t} P_{t-1}(i)^{1-\epsilon}\, di = \int_0^1 P_{t-1}(i)^{1-\epsilon}\,\theta di.$$

Hence, the aggregate price level in t can be written as:

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}.$$

By dividing this equation be P_{t-1}, we get:

$$\begin{aligned}\left(\frac{P_t}{P_{t-1}}\right)^{1-\epsilon} &= \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}, \\ \therefore \Pi_t^{1-\epsilon} &= \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}.\end{aligned}$$

In the *zero-inflation steady state*, where $\Pi_t = 1$, $P_t^* = P_t = P_{t-1}$. So, if we log-linearize around the steady-state, we get:

$$(1-\theta)(1-\epsilon)\frac{P_t^{*-\epsilon}}{P_{t-1}^{1-\epsilon}}P_t^*.p_t^* - (1-\theta)(1-\epsilon)\frac{(P_t^*)^{1-\epsilon}}{P_{t-1}^{2-\epsilon}}P_{t-1}.p_{t-1} = (1-\epsilon)\Pi_t^{-\epsilon}\Pi_t.\pi_t,$$

$$\therefore (1-\theta)(p_t^* - p_{t-1}) = \pi_t. \tag{10}$$

Flexible Prices

Consider this economy under the assumption that there are no nominal rigidities. When the prices are completely flexible (i.e. $\theta = 0$), no dynamic decisions would be made, and we have:

$$Y_t^n(i) = Y_t^n = A_t N_t^n,$$

where n superscripts denote the flexible or natural values of the variables, henceforth. Then, as noted before, all monopolistic firms would choose the same price in each period, which is a fixed mark-up above the marginal cost:

$$P_t = \frac{\epsilon}{(\epsilon-1)}\frac{\partial \Psi_t(Y_t^n)}{\partial Y_t^n}.$$

Marginal cost is given by W_t/A_t. Thus:

$$P_t = \frac{\epsilon}{(\epsilon-1)}\frac{W_t}{A_t}.$$

If we normalize $W_t = 1$, we get:

$$P_t = \frac{\epsilon}{(\epsilon-1)}\frac{1}{A_t}.$$

Under the assumption of a separable utility function, we saw that the intratemporal decision for labor supply must follow the following optimality condition:

$$\frac{(N_t^n)^{\phi}}{(A_t N_t^n)^{-\sigma}} = \frac{(\epsilon-1)}{\epsilon}A_t,$$

$$\therefore (N_t^n)^{\phi+\sigma} = \frac{(\epsilon-1)}{\epsilon}A_t^{1-\sigma}.$$

If we substitute from firms technology for N_t^n, we get:

$$\begin{aligned}\left(\frac{Y_t^n}{A_t}\right)^{\phi+\sigma} &= \frac{(\epsilon-1)}{\epsilon}A_t^{1-\sigma},\\ \therefore \frac{(\epsilon-1)}{\epsilon} &= \left(Y_t^n\right)^{\phi+\sigma} A_t^{-1-\phi},\end{aligned}$$

which, as we expect, is a constant value, independent of t. Moreover, recall that $MC_t^n = \mathscr{M}$ was defined to be equal to the friction-less mark-up (equal to $(\epsilon-1)/\epsilon$). Taking the logarithm of this equation, we get the real marginal cost under flexible prices:

$$mc^n = (\phi+\sigma)\, y_t^n - (1+\phi)\, a_t.$$

On the other hand, the real rate of return in this case is equal to $E_t\left(P_t/\left(Q_t P_{t+1}\right)\right)$, which can be calculated as:

$$\begin{aligned}R_t^n =& E_t\left(\frac{P_t}{Q_t P_{t+1}}\right)\\ =& E_t\left(\frac{P_t}{P_{t+1}}\frac{1}{\beta}\left(\frac{C_{t+1}^n}{C_t^n}\right)^{\sigma}\frac{P_{t+1}}{P_t}\right)\\ =& \frac{1}{\beta}E_t\left(\left(\frac{C_{t+1}^n}{C_t^n}\right)^{\sigma}\right).\end{aligned}$$

In the equilibrium, when $C_t^n = Y_t^n$, after taking the logarithm, we can rewrite this equation as:

$$r_t^n = -\log\beta + \sigma E_t\left(y_{t+1}^n - y_t^n\right).$$

Equilibrium

In equilibrium, the real marginal cost in period $t+k$ of a firm that sets its price in t is

$$MC_{t+k|t} = \frac{W_{t+k}}{A_{t+k}P_{t+k}},$$

which is the same for all firms (no matter at which date they are adjusting their price), and, hence, equal to MC_{t+k}. In logarithms:

$$mc_{t+k|t} = w_{t+k} - p_{t+k} - a_{t+k} = mc_{t+k}.$$

If we substitute this into Equation (9), we have:

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left(\widehat{mc}_{t+k}\right) + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left(p_{t+k} - p_{t-1}\right).$$

Note that, we can expand this equation as:

$$\begin{aligned}
p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left(\widehat{mc}_{t+k}\right) \\
+ (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left(p_{t+k} - p_{t+k-1} + p_{t+k-1} - p_{t+k-2} + \ldots + p_t - p_{t-1}\right) \\
= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left(\widehat{mc}_{t+k}\right) \\
+ (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left(\pi_{t+k} + \pi_{t+k-1} + \ldots + \pi_t\right).
\end{aligned}$$

Thus:

$$\begin{aligned}
p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left(\widehat{mc}_{t+k}\right) + (1-\beta\theta)\left(\pi_t + \beta\theta\pi_t + \beta^2\theta^2\pi_t + \ldots\right) \\
+ (1-\beta\theta)\left(\beta\theta\pi_{t+1} + \beta^2\theta^2\pi_{t+1} + \ldots\right) + \ldots \\
= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left(\widehat{mc}_{t+k}\right) \\
+ (1-\beta\theta)\frac{\pi_t}{(1-\beta\theta)} + \beta\theta(1-\beta\theta)\frac{\pi_{t+1}}{(1-\beta\theta)} + \ldots \\
= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left(\widehat{mc}_{t+k}\right) + \sum_{k=0}^{\infty}(\beta\theta)^k E_t\left(\pi_{t+k}\right). \quad (11)
\end{aligned}$$

If we write this equation for period $t+1$, and take the expectations with respect to the information available at t, by Law of Iterated Expectations, we have:

$$E_t\left(p_{t+1}^* - p_t\right) = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^{k+1}E_t\left(\widehat{mc}_{t+k+1}\right) + \sum_{k=0}^{\infty}(\beta\theta)^{k+1}E_t\left(\pi_{t+k+1}\right),$$

$$\therefore E_t\left(p_{t+1}^* - p_t\right) = (1-\beta\theta)\sum_{k=1}^{\infty}(\beta\theta)^{k}E_t\left(\widehat{mc}_{t+k}\right) + \sum_{k=1}^{\infty}(\beta\theta)^{k}E_t\left(\pi_{t+k}\right).$$

Thus, we may write (11) as:

$$p_t^* - p_{t-1} = \beta\theta E_t\left(p_{t+1}^* - p_t\right) + (1-\beta\theta)\,\widehat{mc}_t + \pi_t$$

Now, if we substitute from (10), we get:

$$\frac{\pi_t}{(1-\theta)} = \beta\theta E_t\left(\frac{\pi_{t+1}}{(1-\theta)}\right) + (1-\beta\theta)\,\widehat{mc}_t + \pi_t,$$

$$\therefore \pi_t = \beta E_t\left(\pi_{t+1}\right) + \frac{(1-\theta)(1-\beta\theta)}{\theta}\widehat{mc}_t. \tag{12}$$

If we solve this equation forward, we get a formula for inflation as a discounted sum of expected real mark-up deviations:

$$\pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}\sum_{k=0}^{\infty}\beta^k E_t\left(\widehat{mc}_{t+k}\right).$$

In the labor market, in equilibrium, noting that

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\epsilon} Y_t,$$

we have:

$$N_t = \int_0^1 \frac{Y_t(i)}{A_t} di$$
$$= \frac{Y_t}{A_t} \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\epsilon} di.$$

If we take logarithm of this equation, we have:

$$n_t = y_t - a_t + d_t,$$

where d_t is a measure of price dispersion across firms. We can show that, around the zero-inflation steady-state, d_t is zero up to a first order approximation. Thus, we may write this equation as:

$$n_t = y_t - a_t. \tag{13}$$

If we substitute from good's market clearing condition into the household's optimality condition for labor supply, we get:

$$w_t - p_t = \sigma y_t + \phi n_t.$$

After substitution for n_t from Equation (13), we get:

$$w_t - p_t = \sigma y_t + \phi (y_t - a_t)$$
$$= (\sigma + \phi) y_t - \phi a_t,$$

and, hence:

$$mc_t = w_t - p_t - a_t$$
$$= \sigma y_t + \phi (y_t - a_t) - a_t$$
$$= (\sigma + \phi) y_t - (1 + \phi) a_t.$$

Noting that, in the zero-inflation steady-state, the logarithm of real marginal cost is equal to the real marginal cost under flexible prices, we can write:

$$\begin{aligned}\widehat{mc}_t =& mc_t - mc \\ =& mc_t - mc^n \\ =& (\sigma + \phi)(y_t - y_t^n).\end{aligned}$$

We define the log-deviation of output from its flexible price counterpart as output gap, and denote it by $\tilde{y}_t \coloneqq y_t - y_t^n$. Thus:

$$\widehat{mc}_t = (\sigma + \phi)\,\tilde{y}_t.$$

We may now write Equation (12) as:

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa \tilde{y}_t, \tag{14}$$

where

$$\kappa \coloneqq \frac{(1-\theta)(1-\beta\theta)}{\theta}(\sigma + \phi).$$

This is the well-known *New-Keynesian Phillips curve.*

On the other hand, we can write Equation (6), in equilibrium as:

$$y_t = E_t(y_{t+1}) - \frac{1}{\sigma}\left[\log\beta + i_t - E_t(\pi_{t+1})\right].$$

Subtracting y_t^n from both sides, we can write this equation as:

$$\begin{aligned}y_t - y_t^n = E_t(y_{t+1}) - E_t\left(y_{t+1}^n\right) + E_t\left(y_{t+1}^n\right) - y_t^n - \frac{1}{\sigma}\left[\log\beta + i_t - E_t(\pi_{t+1})\right], \\ \tilde{y}_t = E_t(\tilde{y}_{t+1}) + E_t\left(\Delta y_{t+1}^n\right) - \frac{1}{\sigma}\left[\log\beta + i_t - E_t(\pi_{t+1})\right], \\ \therefore \tilde{y}_t = E_t(\tilde{y}_{t+1}) - \frac{1}{\sigma}\left[i_t - E_t(\pi_{t+1}) - r_t^n\right], \qquad (15)\end{aligned}$$

where

$$r_t^n \coloneqq -\log\beta + \sigma E_t\left(\Delta y_{t+1}^n\right),$$

is the *natural rate of interest*, as defined before. Equation (15) is referred to as the *dynamic*

IS equation.

If we assume that the effect of nominal rigidities vanishes asymptotically, $\lim_{T\to\infty} E_t\left(\Delta \tilde{y}_{t+T}\right) = 0$, we can solve this equation forward to get:

$$\tilde{y}_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty}\left(r_{t+k} - r^n_{t+k}\right),$$

where $r_t := i_t - E_t\left(\pi_{t+1}\right)$, is the real interest rate (this is the celebrated Fisher equation); i.e. the output gap is proportional to the sum of current and anticipated deviations between the real rate of return and its counterpart under flexible prices.

Equations (14) and (15) are the building blocks of this basic New-Keynesian model (and, as it turns out, majority of New-Keynesian models); the New-Keynesian Phillips curve and dynamic IS equation, together with a policy equation that determines the dynamics of nominal interest rate i, characterize the equilibrium in this economy. Given the exogenous process governing A_t, we know how y_t^n evolves over time. This, in turn, determines the process governing the natural rate of return. Then, Equations (14) and (15), together, determine how the inflation and output gap would change through time.

The process governing i, in turn, depends on how monetary policy is conducted, through the *ad hoc* money demand equation (introduced in the household's section); this is in contrast to classical models where monetary policy is neutral. Here, monetary policy is a key determinant of real changes in the economy. For instance, an equation usually noted in the literature describing how monetary authority reacts to the inflation rate and output gap is the following:

$$i_t = -\log\beta + \alpha_\pi \pi_t + \alpha_y \tilde{y}_t + \nu_t,$$

where α_π and α_y are non-negative coefficients, and v_t is an exogenous component with zero mean. This is the well-known so-called Taylor rule (we discussed why it is not appealing to call this a rule in class).

References

Calvo, Guillermo A., "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 1983, *12* (3), 383–398.