Macroeconomic Theory (ECON 8106)

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Problem Set 4

Due Date: December 2nd, 2016

Please hand in one physical copy per group and write the names of your group members on the first page.

Question 1:

Consider a cash-credit goods economy with preferences given by $\log c_{1t} + \alpha \log c_{2t} + \gamma \log(1 - l_t)$, where c_{1t} denotes consumption of cash goods, c_{2t} denotes credit goods consumption, and l_t denotes labor supply. The resource constraint is

$$g_t + c_{1t} + c_{2t} = l_t.$$

The cash-in-advance constraint is $p_t c_{1t} \leq M_t$, where p_t denotes the price level and M_t denotes cash balances. Households trade money and bonds in the securities market at the beginning of each period. The securities market constraint is

$$M_t + B_t \leq (M_{t-1} - p_{t-1}c_{1t-1}) - p_{t-1}c_{2t-1} + w_{t-1}l_{t-1} + R_{t-1}B_{t-1} - T_{t-1},$$

where B_t denotes the holdings of one-period bonds, R_t is the interest rate, w_t is the wage rate, and T_t is lump-sum taxes.

a. Define an equilibrium in this economy. Remember to include the government's budget constraint.

b. Prove that if there is constant money growth $\mu > \beta$, and if the economy is in a stationary equilibrium from some time τ onwards, it is stationary at all previous dates.

c. Prove that in any stationary equilibrium, if $\frac{T_t}{p_t} = \bar{T}$, sustained deficits imply positive inflation and lower output. Government deficits are defined as the cost of government purchase minus the net revenue from the bonds market and tax revenue.

d. Show that if $R_t > 1$, then the cash in advance constraint must be binding.

e. Now suppose the interest rate is fixed at R_0 in even periods and $R_1 > R_0$ in odd periods. Compute equilibrium allocations in the two types of periods. What can you say about money growth from odd to even periods and from even to odd periods?

Question 2:

Solve Part 2 of the Spring 2011 Macro Prelim.

Question 3:

a. Read Stokey Lucas (1978): "Money and Interest in a Cash-In-Advance Economy"

b. Define an equilibrium in a Cash-Credit economy (as defined in the previous questions) in which the growth of money in any period is governed by a discrete markov process s_t.

c. Show that if you scale all variables by the aggregate supply of money in each period, you can write the problem recursively as in Stokey Lucas. Make explicit any transformations of variables you use.

d. Show that if s_t is i.i.d., then the equilibrium depends only on $E[\frac{1}{g(s')}]$, where g is the growth rate of money, and no other higher moments of $g(s')$.

Question 4:

Show that a cash-credit goods economy is equivalent to an economy in which there is a single good and money is in the utility function. That is, let $c_{1t} + c_{2t} = c_t$, and let real balances at the end of period in a money-in-the-utility-function economy equal to c_{1t}.

Question 5:

Consider the following economy with a representative infinitely lived consumer, a continuum of monopolistic intermediate good producers (of unit mass), competitive final good producers, and a government; in each period t, the economy is subject to an exogenous i.i.d shock that takes values in the set S, according to the probability measure π. Let $s^t \in S^t$ denote the history of shocks up to date t.

Final good producers have access to the following technology to transform intermediate goods to final consumption good:

$$Y = \left[\int_0^1 Y(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}.$$

Each intermediate good producer has access to a technology that transforms one unit of labor into one unit of a differentiated good. Moreover, at each period t, there is a probability $1-\alpha$ that the monopolist can reset its price (according to the history of shocks, s^t). Intermediate good producers in period t discount profits in period $t+k$ at the stochastic discount factor $Q_{t,t+k}$.

Consumer's preferences over streams of consumption and labor are represented by the following expected utility form:

$$\mathbb{E}_0 \sum_t \beta^t \frac{C(s^t)^{1-\sigma}}{1-\sigma} - \frac{N(s^t)^{1+\phi}}{1+\phi} + \frac{(M(s^t)/P(s^t))^{1-\nu}}{1-\nu}$$

where $M(s^t)$ denotes the holdings of nominal money balances.

In each period, the consumer faces a budget constraint of the form

$$P(s^t)C(s^t)+M(s^t)+\sum_{s_{t+1}}Q(s^{t+1}|s^t)B(s^{t+1})\leq W(s^t)N(s^t)+M(s^{t-1})+B(s^t)+T(s^t)+\Pi(s^t)$$

where $P(s^t)$ is the price level, $B(s^t)$ is the Arrow security, $\Pi(s^t)$ is the household's entitlement to nominal profits of intermediate producers, and $T(s^t)$ are the lump-sum transfers.

The timing within each period is as follows: at the beginning of period, the government chooses its monetary policy (through the choice of money supply); exogenous shock s_t is realized; fraction $1-\alpha$ of intermediate good producers reset their prices; and consumers and final good producers make their decisions.

a. Derive the demand for intermediate goods and the price of final goods.

b. What is the optimal pricing policy for the intermediate good producers that reset their price after realization of s_t?

c. Derive the first order conditions for the consumer's problem.