Economics 8106
Macroeconomic Theory
Recitation 4

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Used in Conor Ryan's Recitation

1 A Basic New Keynesian Model

1.1 Motivation

Looking at data on money, output and prices, we see the following marcro and micro evidence

- Macro evidence on the effects of monetary policy shocks:
  - Persistent effects on real variables
  - Slow adjustment of aggregate price level
  - Liquidity effect,i.e. easy credit by monetary expansionary policy results in greater economic activity as businesses and individuals borrow to finance purchases and operations
- Micro evidence: significant price and wage rigidities

This is in conflict with the predictions of classical monetary models. Therefore, our goal here is to build a model that could have the flavor of real effects of monetary policy shocks, which is often called monetary business cycles.

1.2 A Baseline Model with Nominal Rigidities

1.2.1 Environment

There are 4 types of agents in this economy

1. Households: an infinitely-lived representative household whose preferences are represented by

$$\mathbb{E}_0 \sum_t \beta^t U \left[C(s^t), N(s^t), \frac{M(s^t)}{P(s^t)} \right]$$

Household budget constraints:

$$P(s^t)C(s^t) + M(s^t) + \sum_{s_{t+1}} Q(s^{t+1}|s^t)B(s^{t+1}) \le W(s^t)N(s^t) + M(s^{t-1}) + B(s^t) + T(s^t) + \Pi(s^t)$$

2. Final producer: Final good is producing competitively (take prices as given). The final good producer uses a continuum of intermediate goods $Y(i, s^t)$ to produce final output $Y(s^t)$ by a Dizit-Stiglitz CES technology

$$Y(s^t) := \left[\int_0^1 Y(i, s^t)^\theta\right]^{1/\theta}$$

   Here $Y(i, s^t)$ is the output of a differentiated good indexed by i. We assume there is an exogenous continuum of such goods with measure one in the economy.

3. Intermediate producers:

   There is measure one of firms, participating in monopolistic competition (which means they choose their own prices). Imagine there is a Calvo fairy in this economy. At the begining of every period t, the Calvo fairy taps a fraction $(1-\theta)$ of firms randomly, giving them permission to change their prices.

4. Government: The government prints money $M(s^t)$ and supply them to the households. Here we assume that money growth rate is stochastic, i.e.

$$\frac{M(s^t)}{M(s^{t-1})} = \mu(s^t)$$

1.3 Analysis

Final Good Producer

Given prices, final good's producer chooses $Y(i, s^t)$ to solve:

$$\max_{\{Y(i,s^t)\}} \sum_t \sum_{s^t} P(s^t)Y(s^t) - \int_0^1 P(i, s^t)Y(i, s^t)di$$

$$s.t. \quad Y(s^t) = \left[\int_0^1 Y(i, s^t)^\theta di\right]^{1/\theta}$$

which can be rewritten as

$$\max_{\{Y(i,s^t)\}} \sum_t \sum_{s^t} P(s^t) \left[\int_0^1 Y(i, s^t)^\theta di\right]^{1/\theta} - \int_0^1 P(i, s^t)Y(i, s^t)di$$

FOCs:

$$\begin{aligned}
\left[\int_0^1 Y(i,s^t)^\theta di\right]^{\frac{1-\theta}{\theta}} P(s^t)Y(i,s^t)^{\theta-1} &= P(i,s^t) \\
Y(i,s^t)^{\theta-1} &= \frac{P(i,s^t)}{P(s^t)}\left[\int_0^1 Y(i,s^t)^\theta di\right]^{\frac{\theta-1}{\theta}} \\
Y(i,s^t) &= \left[\frac{P(s^t)}{P(i,s^t)}\right]^{\frac{1}{1-\theta}} Y(s^t)
\end{aligned}$$

which is the demand for $Y(i,s^t)$. So

$$Y^d(i,s^t) = \left[\frac{P(s^t)}{P(i,s^t)}\right]^{\frac{1}{1-\theta}} Y(s^t)$$

Also, zero-profit condition implies

$$\begin{aligned}
P(s^t)Y(s^t) - \int_0^1 P(i,s^t)Y(i,s^t)di &= 0 \\
P(s^t)Y(s^t) - \int_0^1 P(i,s^t)\left[\frac{P(s^t)}{P(i,s^t)}\right]^{\frac{1}{1-\theta}} Y(s^t)di &= 0 \\
P(s^t)Y(s^t) - P(s^t)^{\frac{1}{1-\theta}}Y(s^t)\int_0^1 P(i,s^t)^{\frac{\theta}{\theta-1}}di &= 0
\end{aligned}$$

So

$$P(s^t) = \left[\int P(i,s^t)^{\frac{\theta}{\theta-1}}\right]^{\frac{\theta-1}{\theta}}$$

Intermediate Good Producer

Question. *Suppose that prices are flexible in a way that all of the intermediate producers are freely to choose their prices every period, i.e. $\alpha = 0, \forall t$. What is $P(i,s^t)$?*

$$\begin{aligned}
&\max_{P(i,s^t)} P(i,s^t)Y^d(i,s^t) - W(s^t)N(i,s^t) \\
s.t.\quad &Y^d(i,s^t) = \left[\frac{P(s^t)}{P(i,s^t)}\right]^{\frac{1}{1-\theta}} Y(s^t) \\
&Y^d(i,s^t) = N(i,s^t)
\end{aligned}$$

Rewriting

$$\max_{P(i,s^t)} \left[P(i,s^t) - W(s^t)\right]\left[\frac{P(s^t)}{P(i,s^t)}\right]^{\frac{1}{1-\theta}} Y(s^t)$$

FOCs:

$$\begin{aligned}\frac{\theta}{\theta-1}P(s^r)^{\frac{1}{1-\theta}}P(i,s^t)^{\frac{1}{\theta-1}}Y(s^t) &= \frac{1}{\theta-1}W(s^t)P(s^t)^{\frac{1}{1-\theta}}P(i,s^t)^{\frac{1}{\theta-1}-1}Y(s^t) \\ P(i,s^t) &= \frac{1}{\theta}W(s^t)\end{aligned}$$

that is the price of a intermediate good i is a markup of the wage rate on labor.

Calvo Fairy:

Given the demand for intermediate good i, the intermediate producer of good i solves:

$$\begin{aligned}&\max_{P(i,s^t)}\sum_{r=t}^{\infty}\sum_{s^r}\alpha^{r-t}Q(s^r|s^t)\left[P(i,s^t)Y^d(i,s^r)-W(s^r)N(i,s^r)\right] \\ &s.t. \quad Y^d(i,s^r)=\left[\frac{P(s^r)}{P(i,s^t)}\right]^{\frac{1}{1-\theta}}Y(s^r) \\ &\qquad Y^d(i,s^t)=N(i,s^t)\end{aligned}$$

where $Q(s^r|s^t)$ is the stochastic discount factor.

Rewrite this problem:

$$\max_{P(i,s^t)}\sum_{r=t}^{\infty}\sum_{s^r}\alpha^{r-t}Q(s^r|s^t)\left[P(i,s^t)-W(s^r)\right]\left[\frac{P(s^r)}{P(i,s^t)}\right]^{\frac{1}{1-\theta}}Y(s^r)$$

$$\max_{P(i,s^t)}\sum_{r=t}^{\infty}\sum_{s^r}\alpha^{r-t}\left[Q(s^r|s^t)P(s^r)^{\frac{1}{1-\theta}}P(i,s^t)^{\frac{\theta}{\theta-1}}Y(s^r)-W(s^r)P(s^r)^{\frac{1}{1-\theta}}P(i,s^t)^{\frac{1}{\theta-1}}Y(s^r)\right]$$

FOCs:

$$P(i,s^t)=\frac{\sum_{r=t}^{\infty}\sum_{s^r}\alpha^{r-t}Q(s^r|s^t)W(s^r)P(s^r)^{\frac{1}{\theta-1}}Y(s^r)}{\theta\sum_{r=t}^{\infty}\sum_{s^r}\alpha^{r-t}Q(s^r|s^t)P(s^r)^{\frac{1}{\theta-1}}Y(s^r)} \tag{1}$$

Households

Given prices, HH solves:

$$\begin{aligned}&\max\sum_{t}\sum_{s^t}\beta^t\pi_t(s^t)U\left[C(s^t),N(s^t),\frac{M(s^t)}{P(s^t)}\right] \\ &s.t. \quad P(s^t)C(s^t)+M(s^t)+\sum_{s_{t+1}}Q(s^{t+1}|s^t)B(s^{t+1}) \\ &\qquad \le W(s^t)N(s^t)+M(s^{t-1})+B(s^t)+T(s^t)+\Pi(s^t) \\ &\qquad \lim_{T\to\infty}\mathbb{E}_t(B(s^{T+1}))\ge 0 \\ &\qquad B_0(s_0),M_{-1}(s^{-1})\text{ given}\end{aligned}$$

FOCs for HHs:

$$\begin{aligned}
\beta^t \pi(s^t) U_c(s^t) &= \lambda(s^t) P(s^t) \\
\beta^t \pi(s^t) U_n(s^t) &= -\lambda(s^t) W(s^t) \\
\lambda(s^t) &= \beta^t \pi_t(s^t) \frac{U_m(s^t)}{P(s^t)} + \sum_{s_{t+1}|s^t} \lambda(s^{t+1}) \\
\lambda(s^t) Q(s^{t+1}|s^t) &= \lambda(s^{t+1})
\end{aligned}$$

Divide the first two equations, and substitute in $\lambda(s^t), \lambda(s^{t+1})$ from the first equation to other equations and simplify, we get

$$-\frac{U_n(s^t)}{U_c(s^t)} = \frac{W(s^t)}{P(s^t)} \tag{2}$$

$$\frac{U_c(s^t)}{P(s^t)} - \frac{U_m(s^t)}{P(s^t)} = \beta \sum_{s_{t+1}|s^t} \pi(s^{t+1}|s^t) \frac{U_c(s^{t+1})}{P(s^{t+1})} \tag{3}$$

$$Q(s^{t+1}|s^t) = \beta \pi(s^{t+1}|s^t) \frac{U_c(s^{t+1})}{U_c(s^t)} \frac{P(s^t)}{P(s^{t+1})} \tag{4}$$

Question. *What are the government budget constraint and market clearance conditions?*

$$\begin{aligned}
M(s^t) &= \mu(s^t) M(s^{t-1}) \\
T(s^t)) &= M(s^t) - M(s^{t-1})
\end{aligned}$$

$$\begin{aligned}
C(s^t) &= Y(s^t) \\
N(s^t) &= \int N(i, s^t) di \\
B(s^{t+1}) &= 0
\end{aligned}$$

1.4 Log-linearization

Assumption 1. *Assume now that the preference is*

$$U = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} + \frac{(M_t/P_t)^{1-\nu}}{1-\nu}$$

The FOCs of HHs become

$$\begin{aligned}
\frac{N(s^t)^{\phi}}{C(s^t)^{-\sigma}} &= \frac{W(s^t)}{P(s^t)} && (5)\\
\frac{C(s^t)^{-\sigma}}{P(s^t)} - \frac{(M(s^t)/P(s^t))^{-\nu}}{P(s^t)} &= \beta \sum_{s_{t+1}|s^t} \pi(s^{t+1}|s^t)\frac{C(s^{t+1})^{-\sigma}}{P(s^{t+1})} && (6)\\
Q(s^{t+1}|s^t) &= \beta\pi(s^{t+1}|s^t)\frac{C(s^{t+1})^{-\sigma}}{C(s^t)^{-\sigma}}\frac{P(s^t)}{P(s^{t+1})} && (7)\\
Q(s^r|s^t) &= \beta^{r-t}\pi(s^r|s^t)\frac{C(s^r)^{-\sigma}}{C(s^t)^{-\sigma}}\frac{P(s^t)}{P(s^r)} && (8)
\end{aligned}$$

Take log on both sides of equation 5, we have

$$\phi \log N_t + \sigma \log C_t = \log W_t - \log P_t$$

Linearize this equation with respect to the steady state values, we have

$$\phi\left[\log N_{ss} + \frac{1}{N_{ss}}\left(N_t - N_{ss}\right)\right] + \sigma\left[\log C_{ss} + \frac{1}{C_{ss}}\left(C_t - C_{ss}\right)\right] = \\ \log W_{ss} + \frac{1}{W_{ss}}\left(W_t - W_{ss}\right) - \left[\log P_{ss} + \frac{1}{P_{ss}}\left(P_t - P_{ss}\right)\right]$$

Define

$$a_t = \frac{1}{a_{ss}}\left(A_t - A_{ss}\right)$$

then we have

$$\phi n_t + \sigma c_t = w_t - p_t$$

For money demand, substitute 7 into 6, we have

$$\begin{aligned}
\frac{(M(s^t)/P(s^t))^{-\nu}}{P(s^t)} &= \left[1 - Q(s^{t+1}|s^t)\right]\frac{C(s^t)^{-\sigma}}{P(s^t)}\\
\frac{(M_t/P_t)^{-\nu}}{C_t^{-\sigma}} &= 1 - e^{-i_t} \approx i_t
\end{aligned}$$

where $i_t = -\log Q_{t,t+1}$

Log-linearize this equation, we have

$$\begin{aligned}
m_t - p_t - \frac{\sigma}{\nu}c_t &= -\frac{1}{\nu(e^i - 1)}i_t + \frac{i}{\nu(e^i - 1)}\\
m_t - p_t &\approx \frac{\sigma}{\nu}c_t - \eta i_t
\end{aligned}$$

Log-linearize the Euler's equation, we have

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \Pi_{t+1} - \rho\right)$$

where $\rho = -\log\beta, \Pi_{t+1} = P_{t+1}/P_t$

Substitute 7 to 1 the equation of price for intermediate good:

$$\begin{aligned}
P(i,s^t) &= \frac{\sum_{r=t}^{\infty}\sum_{s^r}\alpha^{r-t}\beta^{r-t}\pi(s^r|s^t)\frac{C(s^r)^{-\sigma}}{C(s^t)^{-\sigma}}\frac{P(s^t)}{P(s^r)}W(s^r)P(s^r)^{\frac{1}{\theta-1}}Y(s^r)}{\theta\sum_{r=t}^{\infty}\sum_{s^r}\alpha^{r-t}\beta^{r-t}\pi(s^r|s^t)\frac{C(s^r)^{-\sigma}}{C(s^t)^{-\sigma}}\frac{P(s^t)}{P(s^r)}P(s^r)^{\frac{1}{\theta-1}}Y(s^r)} \\
\frac{P(i,s^t)}{P(s^{t-1})} &= \frac{\sum_{r=t}^{\infty}\sum_{s^r}(\beta\alpha)^{r-t}\pi(s^r|s^t)C(s^r)^{1-\sigma}W(s^r)P(s^r)^{\frac{2-\theta}{\theta-1}}}{\theta\sum_{r=t}^{\infty}\sum_{s^r}(\beta\alpha)^{r-t}\pi(s^r|s^t)C(s^r)^{1-\sigma}P(s^r)^{\frac{2-\theta}{\theta-1}}}\frac{1}{P(s^{t-1})}
\end{aligned}$$

So

$$\begin{aligned}
&\frac{P(i,s^t)}{P(s^{t-1})}\sum_{r=t}^{\infty}\sum_{s^r}(\beta\alpha)^{r-t}\pi(s^r|s^t)C(s^r)^{1-\sigma}P(s^r)^{\frac{2-\theta}{\theta-1}} \\
&= \frac{1}{\theta}\sum_{r=t}^{\infty}\sum_{s^r}(\beta\alpha)^{r-t}\pi(s^r|s^t)C(s^r)^{1-\sigma}W(s^r)P(s^r)^{\frac{2-\theta}{\theta-1}}\frac{1}{P(s^{t-1})} \qquad (9)
\end{aligned}$$

In zero-inflation steady states, we must have

$$\begin{aligned}
\Pi_t &= \frac{P(i,s^t)}{P(s^{t-1})} = \frac{P(i,s^t)}{P(s^t)} = \frac{P(i,s^t)}{P(s^r)} = 1 \\
Y(s^r) &= Y(s^t) \\
Q(s^r|s^t) &= \beta^{r-t} \\
W(s^r) &= \theta P
\end{aligned}$$

Linearize eq 9 around the zero-inflation steady states (w.r.t. $P(i,s^t), P(s^{t-1}), P(s^r), C(s^r), W(s^r)$)

$$\begin{aligned}
LHS &= \sum_{r=t}^{\infty}(\beta\alpha)^{r-t}C^{1-\sigma}P^{\frac{2-\theta}{\theta-1}} + \frac{1}{P}\mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}C^{1-\sigma}P^{\frac{2-\theta}{\theta-1}}(P(i,s^t)-P) \\
&\quad -\frac{P}{P^2}\mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}C^{1-\sigma}P^{\frac{2-\theta}{\theta-1}}(P(s^{t-1})-P) \\
&\quad +\mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}C^{1-\sigma}\left(\frac{2-\theta}{\theta-1}\right)P^{\frac{1}{\theta-1}}(P(s^r)-P) \\
&\quad +\mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}(1-\sigma)C^{-\sigma}P^{\frac{2-\theta}{\theta-1}}(C(s^r)-C)
\end{aligned}$$

So

$$\begin{aligned}
LHS &= \sum_{r=t}^{\infty}(\beta\alpha)^{r-t}C^{1-\sigma}P^{\frac{2-\theta}{\theta-1}} + \mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}C^{1-\sigma}P^{\frac{2-\theta}{\theta-1}}p_t(i) \\
&\quad -\mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}C^{1-\sigma}P^{\frac{2-\theta}{\theta-1}}p_{t-1} \\
&\quad +\mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}C^{1-\sigma}\left(\frac{2-\theta}{\theta-1}\right)P^{\frac{\theta}{\theta-1}}p_r \\
&\quad +\mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}(1-\sigma)C^{1-\sigma}P^{\frac{2-\theta}{\theta-1}}c_r
\end{aligned}$$

Similarly,

$$\begin{aligned} RHS &= \frac{1}{\theta}\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}C^{1-\sigma}P^{\frac{2-\theta}{\theta-1}}W_{ss} \\ &-\frac{1}{\theta}\mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}C^{1-\sigma}P^{\frac{2-\theta}{\theta-1}}W_{ss}\frac{1}{P}p_{t-1} \\ &+\frac{1}{\theta}\mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}C^{1-\sigma}\left(\frac{2-\theta}{\theta-1}\right)P^{\frac{\theta}{\theta-1}}W_{ss}\frac{1}{P}p_r \\ &+\frac{1}{\theta}\mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}(1-\sigma)C^{1-\sigma}P^{\frac{2-\theta}{\theta-1}}W_{ss}\frac{1}{P}c_r \\ &+\mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}C^{1-\sigma}P^{\frac{2-\theta}{\theta-1}}w_r \end{aligned}$$

So

$$\begin{aligned} RHS &= \sum_{r=t}^{\infty}(\beta\alpha)^{r-t}C^{1-\sigma}P^{\frac{2-\theta}{\theta-1}} \\ &-\mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}C^{1-\sigma}P^{\frac{2-\theta}{\theta-1}}p_{t-1} \\ &+\mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}C^{1-\sigma}\left(\frac{2-\theta}{\theta-1}\right)P^{\frac{\theta}{\theta-1}}p_r \\ &+\mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}(1-\sigma)C^{1-\sigma}P^{\frac{2-\theta}{\theta-1}}c_r \\ &+\mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}C^{1-\sigma}P^{\frac{2-\theta}{\theta-1}}w_r \end{aligned}$$

Equating the LHS and RHS, we have that

$$\begin{aligned} \mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}C^{1-\sigma}P^{\frac{2-\theta}{\theta-1}}p_t(i) &= \mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}C^{1-\sigma}P^{\frac{2-\theta}{\theta-1}}w_r \\ \mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}p_t(i) &= \mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}w_r \\ p_t(i) &= (1-\beta\alpha)\mathbb{E}_t\sum_{r=t}^{\infty}(\beta\alpha)^{r-t}w_r \end{aligned}$$